Robust feedback switching control: dynamic programming and viscosity solutions and ergodicity

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We consider a robust switching control problem. The controller only observes the evolution of the state process, and thus uses feedback (closed-loop) switching strategies, a nonstandard class of switching controls introduced in this paper. The adverse player (nature) chooses open-loop controls that represent the so-called Knightian uncertainty, i.e., misspecifications of the model. The (half) game switcher versus nature is then formulated as a two-step (robust) optimization problem. We develop the stochastic Perron's method in this framework, and prove that it produces a viscosity subsolution and supersolution to a system of HJB variational inequalities, which envelop the value function. Together with a comparison principle, this characterizes the value function of the game as the unique viscosity solution to the HJB equation, and shows as a by-product the dynamic programming principle for the robust feedback switching control problem.

Next, we analyze the asymptotic behavior for a system of fully nonlinear parabolic and elliptic quasi variational inequalities. These equations are related to robust switching control problems introduced in [3]. We prove that, as time horizon goes to infinity (resp. discount factor goes to zero) the long run average solution to the parabolic system (resp. the limiting discounted solution to the elliptic system) is characterized by a solution of a nonlinear system of ergodic variational inequalities. Our results hold under a dissipativity condition and without any non degeneracy assumption on the diffusion term. Our approach uses mainly probabilistic arguments and in particular a dual randomized game representation for the solution to the system of variational inequalities.

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